

1.1 Analytical Reasoning: A Tool for Life

Introduction

One of the primary goals of this text is to foster and promote your analytical reasoning skills. Each of us has these skills and uses them in all aspects of daily life without consciously being aware of it. Determining the time at which you need to leave home to arrive at an appointment or a meeting is a simple example; calculating how much money you need to earn in a given period of time in order to pay your bills is another.

Analytical Reasoning: A Tool for Life

Analytical reasoning is the primary tool used in solving problems of every kind, including math problems. While all students make use of these skills, many students have difficulty applying these skills to those dreaded “word problems” in math classes and to math problems in general. This course of study has been designed to facilitate the use of your analytical reasoning skills in a variety of settings and applications so that you gradually develop and deepen these skills as you proceed through the textbook.

By the end of this course, you will be able to look back and recognize this process more clearly; for the present, please be encouraged! Solving problems and investigating mathematical relationships and connections can and will be quite enjoyable as you progress through this text. In the end, the development of your analytical reasoning skills will enable you to become more effective problem solvers in all areas of your life, not only in mathematics. (Incidentally, a number of my former students have reported that their reasoning skills improved as a result of the materials in this text.)

In this section, we will outline some basic problem-solving strategies that will be helpful to you as you progress through the text. Many opportunities to apply these skills and strategies are provided throughout the text in the forms of both *Opportunities to Exercise your Analytical Reasoning Skills* and *Discovery Moments*; other opportunities are provided in the *Exercises and Projects for Fun and Profit* at the end of each textbook section.

Problem-solving Strategies

Solving problems is something that all of us do throughout our lifetimes. It can be a very rewarding activity, especially when the situation requires some measure of creativity in order to find a solution. In order to facilitate your use of various problem-solving strategies throughout this course, I have provided an outline of the most widely used strategies below. This outline is

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certainly not exhaustive, but it does include the strategies that are most commonly employed in the processes of problem solving.

The strategies are loosely divided into three sections:

- fundamental
- algebraic
- general purpose

The fundamental strategies are those that are necessary and valuable in almost any situation while the algebraic strategies apply primarily to problems in which there are some unknown quantities in relationship with each other. The general purpose strategies are also valuable in many situations, especially those that are not algebraic in nature. I trust that you will find these to be helpful and instructive.

fundamental strategies

- clearly define the problem in your own words
- construct a diagram whenever possible
- clearly state the question that you will attempt to answer
- identify the given facts and record them in a list or table
- identify what it is that you need to know in order to answer your stated question

algebraic strategies (if the problem invites an algebraic approach)

- identify the important features of the problem
- label the unknown features with variables
- apply relevant formulas to the data, where applicable (e.g., $\text{distance} = \text{rate} \cdot \text{time}$)
- construct mathematical equation(s) to represent the stated facts
- solve the equation(s) to obtain a solution

general purpose strategies

- look for patterns in the data
- use trial and error, sometimes referred to as *guess and check*
- eliminate the impossible; then only the possible remains
- take the path of least resistance
- use analytical reasoning to narrow the field of possibilities
- gather all relevant data before jumping to a conclusion
- relate the given problem to other similar problems that you have previously encountered

In the next section, *Math Classics and Applied Reasoning*, a number of these strategies will be illustrated as we examine some classic recreational math problems from days gone by and some new versions on which you can apply your skills. Meanwhile, the *Creative Project* below will allow you the opportunity to try your hand at problem solving.

Exercises and Projects for Fun and Profit

Creative Project:

Pascal's Famous Triangle

CALCULATOR HELP—RAISING A NUMBER TO A POWER

To raise a number to a power on most **scientific calculators** (including the one that is accessible via your computer's Start/Programs/Accessories menu), first enter the number (also known as the base), then press the button labeled y^x (sometimes labeled x^y), then enter the desired power (also known as the exponent), and press the "=" key.

For example, to raise 5 to the 3rd power (i.e., 5 cubed, or 5^3), enter the base value of 5, press the y^x button (or the x^y button), enter the exponent value of 3, and press the "=" key. The result, 125, will then appear on the screen.

If you have a **graphing calculator**, your calculator most likely uses a button labeled with the caret symbol, \wedge , in place of the button labeled y^x on scientific calculators. To raise a number to a power, first enter the number (also known as the base), then press the button labeled \wedge , then enter the desired power (also known as the exponent), and press Enter.

For example, to raise 5 to the 3rd power (i.e., 5 cubed, or 5^3), enter the base value of 5, press the \wedge button, enter the exponent value of 3, and press Enter. The result, 125, will then appear on the screen.

To display bases and exponents in a Word document, there are two options:

1. **The simplest method** is to use the **caret key**, \wedge , to indicate the use of an exponent. For example, 5 raised to the 3rd power may be typed into your Word document as 5^3 . This is universally understood in mathematics (and computer programming).

Note: The caret key, \wedge , is generally located above one of the numeric keys (e.g., the numeral 6) on your keyboard; use the **Shift** key to access it.

2. A more complicated method (but with a more professional appearance) is to use the Equation Editor that is usually available within Microsoft Word.

PASCAL'S FAMOUS TRIANGLE

The array of numbers below shows the first six rows of *Pascal's triangle*. While its historical origins are unknown, the triangle is named in honor of the French mathematician, Blaise Pascal (1623–1662), who popularized its use.

1						row #0	sum of entries = 1 = 2⁰
	1	1				row #1	sum of entries = 2 = 2¹
		1	2	1		row #2	sum of entries = 4 = 2²
	1	3	3	1		row #3	sum of entries = 8 = 2³
	1	4	6	4	1	row #4	sum of entries = 16 = 2⁴
1	5	10	10	5	1	row #5	sum of entries = 32 = 2⁵

Note: **row #3** begins with: 1, 3, ...; **row #4** begins with: 1, 4, ... ; and so on.

Each row of the triangle begins and ends with a 1. A number in the middle portion of a row is obtained by adding the entries diagonally to the left and diagonally to the right in the row immediately above. For example, in the bottom row shown above (**row #5**), the first 5 is the sum of the 1 that is diagonally above and to its left and the 4 that is diagonally above and to its right. Continuing with this pattern, the next row would begin with 1, and then 1+5 would produce a 6 as the second entry, 5 + 10 would produce 15 as the third entry, and so on. Hence the next row (**row #6**) would consist of the entries:

1 6 15 20 15 6 1

The *sums of the entries* in the rows of the triangle also display a pattern. A *sum of entries* in a row is found by adding together the entries; for example, the sum in **row #3** is 1+3+3+1=8.

Pascal's triangle appears in the study of many topics in mathematics, including algebra, counting techniques, and probability.

Assignments:

For each assignment, please answer the following four questions:

<i>question</i>	<i>Assignment #1</i>	<i>Assignment #2</i>	<i>Assignment #3</i>	<i>Assignment #4</i>
1. Complete rows through:	Row #9	Row #11	Row #13	Row #15
2. Find the sum of entries in:	Row #10	Row #12	Row #14	Row #16
3. Find the sum of entries in:	Row #20	Row #22	Row #24	Row #26
4. See below	See below	See below	See below	See below

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Note: You don't necessarily have to complete the rows of the triangle through an indicated row in **Questions #2 and #3**; you may instead determine the sum of the entries in an indicated row based on your observation of the pattern of sums shown in the project introduction.

4. (**Applies to all four assignments above.**) Powers of a number are obtained by multiplying the number by itself a repeated number of times. For example:

$$5^2 = 5 \cdot 5 = 25 \quad \text{and} \quad 5^3 = 5 \cdot 5 \cdot 5 = 125$$

By definition, $a^0 = 1$ for all non-zero values of a . Evaluate the first five powers of 11, beginning with 0.

$$\begin{aligned} 11^0 &= 1 \\ 11^1 &= 11 \\ 11^2 &= \\ 11^3 &= \\ 11^4 &= \end{aligned}$$

Note: If you feel the need, please review the section entitled *Calculator Help—Raising a Number to a Power* that immediately precedes this project.

Notice what you obtain in comparison with Pascal's triangle, i.e., notice how the digits in each power of 11 correspond to a row of entries in the triangle. For examples, $11^2=121$ and **row #2** in the triangle contains the entries 1 2 1; $11^3=1331$ and **row #3** in the triangle contains the entries 1 3 3 1. The entries in each row, when considered together as a single number, correspond to a power of 11; in fact, the row number indicates the power of 11 that the entries in the row represent.

$$\begin{array}{ll} \text{row \#2:} & 1 \ 2 \ 1 & 11^2=121 \\ \text{row \#3:} & 1 \ 3 \ 3 \ 1 & 11^3=1331 \end{array}$$

- a) Does this correspondence continue in the fifth row and in the following rows?
- b) Please explain why the correspondence continues or why the correspondence doesn't continue based on your answer to the preceding question.
