

## 1.2 Math Classics and Applied Reasoning

### *Introduction*

Mathematics provides a wealth of opportunities to apply your analytical reasoning skills. In particular, the branch of mathematics known as *Recreational Mathematics* is a source of many creative and challenging problems that require various problem-solving techniques such as those mentioned in the preceding section of the text.

In this section, we will explore a few of these math classics and then provide you with opportunities to explore additional classics in the *Exercises and Projects for Fun and Profit* at the end of this section. Each of these math classics has become a classic in part because the solution, even though simple in essence, is not immediately obvious. Finding the solution requires analytical reasoning, persistence and creative thinking—three hallmarks of mathematical endeavor.

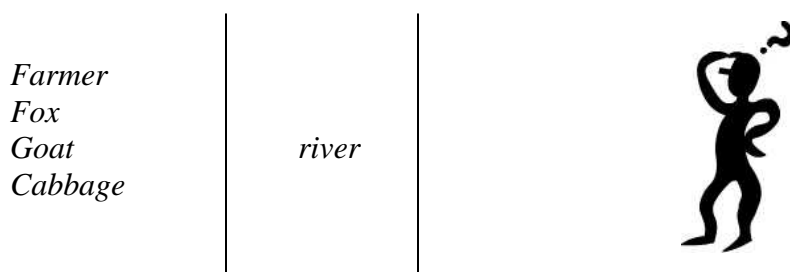
### *Crossing the River: The Story of the Fox, the Goat and the Cabbage*

This story problem has been presented in various guises but perhaps the most familiar is that of the farmer who needs to cross a river with a fox, a goat and a cabbage. His objective is to safely transport them across the river under the following conditions:

- His small boat can carry only himself and one of his possessions (fox, goat or cabbage) at a time.
- The fox will eat the goat if they are left alone together.
- The goat will eat the cabbage if they are left alone together.

The question is: how can the farmer safely convey his fox, goat and cabbage across the river?

### *Strategy: Construct a diagram*



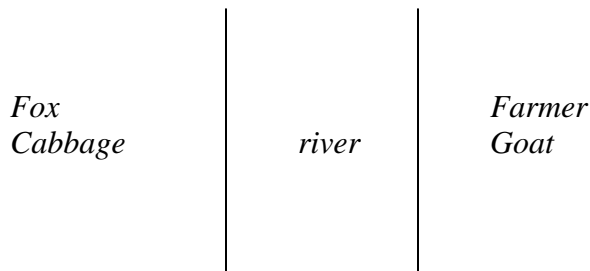
***Math Is Everywhere! Explore and Discover It!***  
***1.2 Math Classics and Applied Reasoning***

***Strategy: Trial and error***

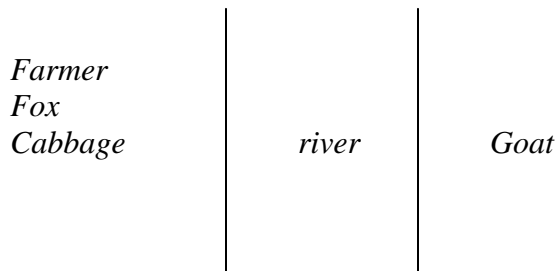
Since the problem invites experimentation, simply choose the fox, the goat or the cabbage to accompany the farmer and see where that choice leads:

- If the farmer takes the fox, then the goat and cabbage will be left together and the goat will eat the cabbage.
- If the farmer takes the goat, then the fox and cabbage will be left together; this is not a problem.
- If the farmer takes the cabbage, then the fox and goat will be left together and the fox will eat the goat.

***Consequently, the farmer must first take the goat across the river.***



The farmer then returns to the fox and the cabbage and must choose which to take next.



Continuing with the trial and error approach, if the farmer takes the fox across the river and leaves it with the goat, the fox will eat the goat; similarly, if the farmer takes the cabbage across the river and leaves it with the goat, the goat will eat the cabbage.

***This presents a veritable dilemma!***

***Math Is Everywhere! Explore and Discover It!***  
***1.2 Math Classics and Applied Reasoning***

**\*\*\*\*\* An Opportunity to Exercise your Analytical Reasoning Skills \*\*\*\*\***



What shall the farmer do in this situation???

\*\*\*\*\*

***Strategy: Eliminate the impossible; then only the possible remains.***

The farmer must bring either the fox or the cabbage across the river and leave it there; but since he can leave neither the fox nor the cabbage with the goat, those choices are impossible while the goat is there.

The only possible remaining course of action is to bring either the fox or the cabbage across and then remove the goat by taking it back across the river with him to the original side.

He can then leave the goat on the original side while he brings the remaining item (either fox or cabbage) across the river. Then the fox and the cabbage will both be together again and the farmer may make one last trip back across the river to pick up the goat and convey it to its destination as well. Success at last! ☺

**\*\*\*\*\* Another Opportunity to Exercise your Analytical Reasoning Skills \*\*\*\*\***



The unanswered question is: After conveying the goat across the river and returning to the fox and the cabbage, which should the farmer choose to take next—the fox or the cabbage?

If you're not sure, you may find the answer to this question at the end of this section on p.21 following the *Exercises and Projects for Fun and Profit*.

\*\*\*\*\*

***The Prize Money and its Effects***

There are many variations of this story problem but the essence is that an additional sum of money will affect the status quo. In this version, two friends have some ten-dollar bills in their pockets and each has less than fifty dollars. Each of them has entered a contest in which the prize is \$30.

The first friend declares: If I win the contest, I'll be twice as rich as you.

The second replies: Yes, but if I win the contest, I'll be five times as rich as you.

How much money does each of them have in their pockets?

## ***Math Is Everywhere! Explore and Discover It!***

### ***1.2 Math Classics and Applied Reasoning***

***Strategy: This type of problem lends itself to an algebraic approach in which you:***

- ***identify the important features of the problem***
- ***label the unknown features with variables***
- ***apply relevant formulas to the data, where applicable (e.g., distance = rate · time)***
- ***construct mathematical equation(s) to represent the stated facts***
- ***solve the equation(s) to obtain a solution***

In this case, the important facts are that the two people involved have some ten-dollar bills and each has less than five of these bills. Additional important facts are presented in the two friends' statements. Since the ultimate goal in this problem is to determine how much money each friend has, it makes good sense to label their current amounts of money as follows:

$a$  = the amount of money that the first friend has at the beginning

$b$  = the amount of money that the second friend has at the beginning

The next step is to construct simple equations that represent the statements concerning the effects of the prize money. If the first friend wins the \$30, then adding the prize money to his current amount will result in twice the current amount of the second friend. If the second friend wins, then adding the prize money to his current amount will result in five times the current amount of the first friend. Using the variables chosen above, we can write the equations:

$$a + 30 = 2b$$

$$b + 30 = 5a$$

Finally, the objective is to solve these equations in some manner so that we may determine the values of  $a$  and  $b$ . Since this is a system of simultaneous equations, i.e., a set of equations in which the solution values for the variables must satisfy both equations, we may use the process of eliminating a variable in order to arrive at a solution. First, we'll rearrange the equations by adding and subtracting terms in each equation to obtain:

$$a - 2b = -30$$

$$-5a + b = -30$$

We may then multiply the first equation by five and add the result to the second equation in order to eliminate the variable  $a$ :

$$5a - 10b = -150$$

$$-5a + b = -30$$

By addition,

$$-9b = -180$$

$$b = 20$$

This means that the only value that  $b$  can have is \$20. Substituting this value back into  $a + 30 = 2b$  yields the equation:

$$a + 30 = 40$$

***Math Is Everywhere! Explore and Discover It!***  
***1.2 Math Classics and Applied Reasoning***

From this we can determine that the solution value for  $a$  is \$10.

The algebraic approach to this problem is quite effective and guarantees that we will obtain a result of some sort.

***Alternative strategy: Trial and error***

We could also have used the strategy of trial and error to construct a list of possible values for  $a$  and  $b$  to see which ones fit the conditions of the problem. With regard to the first friend's statement, we have:

$a$	$b$	$a+30$	$2b$
<b>10</b>	10	<b>40</b>	20
20	<b>20</b>	50	<b>40</b>
<b>30</b>	<b>30</b>	<b>60</b>	<b>60</b>
40	40	70	80

Since the value of  $a + 30$  must equal the value of  $2b$ , the only possible solutions are:  $a=10, b=20$  and  $a=30, b=30$ .

If we consider the other friend's statement, we have:

$a$	$b$	$b+30$	$5a$
<b>10</b>	10	40	<b>50</b>
20	<b>20</b>	<b>50</b>	100
30	30	60	150
40	40	70	200

In this case, since the value of  $b+30$  must equal the value of  $5a$ , the only possible solution is  $a=10, b=20$ . Consequently, since both equations have to be satisfied at the same time, the only mutual solution to the problem is  $a=10, b=20$ .

Since the number of values to be tested is small, trial and error is an acceptable strategy for this problem; however, if there were many possible values for the variables, then this strategy would become quite tedious and the algebraic approach would be far more efficient.

\*\*\*\*\*

***As seen above, there is often more than one way to approach a problem in an attempt to obtain a solution. In this course of study, I would encourage you to use your analytical reasoning abilities and your critical thinking skills to approach our exercises from various directions, whichever seem most suitable or convenient to you at the time.***

\*\*\*\*\*

**Math Is Everywhere! Explore and Discover It!**  
**1.2 Math Classics and Applied Reasoning**

*A Math Crossword Puzzle (actually a Cross-number Puzzle)*

The crossword puzzle has been a standard fixture in literature for many years. In the following mathematical version, each entry both across and down is a positive whole number.

The dilemma, as is usually the case with a crossword puzzle, is that many clues allow for more than one possible answer. This means that you may have to temporarily withhold your decision concerning an answer to a clue while you investigate other adjoining clues and their possible answers.

As an example of this process, we'll solve the following cross-number puzzle; the clues are provided below:

1		2	
		3	
4	5		
	6		

<p><b>Across:</b></p> <p>1. The square of a positive integer (i.e., a whole number multiplied by itself)</p> <p>3. A prime number less than 100 (a prime is divisible only by 1 and itself)</p> <p>4. A multiple of a gross (i.e., a multiple of 144: 144, 288, 432, 576, ...)</p> <p>6. A number that is both a square and a cube</p>	<p><b>Down:</b></p> <p>1. The cube of a positive integer (i.e., a whole number multiplied by itself twice; e.g., <math>9 \cdot 9 \cdot 9 = 729</math>)</p> <p>2. A leap year in the 20<sup>th</sup> century (a leap year is a year that is evenly divisible by 4)</p> <p>5. The number of trombones in an old popular show tune (<i>The Music Man</i>, 1957)</p>
--	--

\*\*\*\*\* *An Opportunity to Exercise your Analytical Reasoning Skills* \*\*\*\*\*  
 ☺ ☺ ☺

Where does one begin in a situation like this? This is a seemingly perplexing dilemma. On which clue would you focus first in order to begin solving the puzzle?

\*\*\*\*\*

***Math Is Everywhere! Explore and Discover It!***  
***1.2 Math Classics and Applied Reasoning***

***Strategy: Take the path of least resistance.***

If you chose “5 Down” or “6 Across,” then you’re on the right track—these clues appear to have only a very limited number of possible solutions. While the show tune is not contemporary, it has earned a place of recognition in our American culture and will hopefully be recognizable to you. If not, you may focus on the other clue that may have only one solution, viz., “6 Across.” Making a list of the squares and cubes less than one hundred will surely provide a solution to this clue. While taking the path of least resistance may seem like an obvious problem-solving strategy, it is often overlooked by students as they consider the various aspects of a problem.

***Strategy: Use analytical reasoning to narrow the field of possibilities.***

Assuming that you have discovered the answers to “5 Down” (76 trombones) and “6 Across,” the next step is to decide which of the other clues are most likely to yield solutions. At this point in the puzzle, there don’t appear to be any clear choices to make. Consequently, you’ll need to resort to solving two or more clues simultaneously and then use analytical reasoning to narrow the field of potential choices.

The clue for “1 Across” allows for a number of possibilities as does the clue for “1 Down.” While the number of three-digit cubes is rather limited (5), the number of three-digit squares is quite lengthy (21). This does make things complicated!

***\*\*\*\*\* Another Opportunity to Exercise your Analytical Reasoning Skills \*\*\*\*\****



Since no solution is immediately forthcoming from the clues for “1 Across” and “1 Down,” where will you look next?

\*\*\*\*\*

Yes, that’s it! ☺ The clue for “2 Down” offers a valuable piece of information, viz., the leap year must begin with 1 since the year is in the twentieth century. This small scrap of information is actually quite valuable! Since “2 Down” must begin with 1, then “1 Across” must end with 1. This immediately limits the choices for “1 Across” to the following square numbers: 121, 361, 441, 841 and 961 (i.e., the squares of 11, 19, 21, 29 and 31).

In turn, this limits the choices for “1 Down” to those cubes that begin with one of the digits 1, 3, 4, 8, and 9. Since the only three-digit numbers that are cubes are 125, 216, 343, 512 and 729, we can eliminate 216, 512 and 729 from consideration. This leaves only 125 and 343 as possible solutions for “1 Down.”

***Math Is Everywhere! Explore and Discover It!***  
***1.2 Math Classics and Applied Reasoning***

***Strategy: Gather all relevant data before jumping to a conclusion.***

Before jumping to the conclusion that the answer to “1 Across” is 121 and that the answer to “1 Down” is 125, we need to be sure that we have gathered all the relevant facts concerning this mutual solution. The primary question of importance is: Are there any other possible solutions under the given circumstances?

An inspection of the lists of squares and cubes given above reveals that there is another combination of a square and a cube that satisfies both pertinent clues: 361 and 343.

***\*\*\*\*\* Another Opportunity to Exercise your Analytical Reasoning Skills \*\*\*\*\****



Since either of the proposed solutions seems to work equally well, we must search for yet another clue that will lead to the elimination of one of the possible solution pairs for “1 Across” and for “1 Down.” What is your next task in your pursuit of a solution to the puzzle?

\*\*\*\*\*

Yes, that’s it! You need to investigate the multiples of a gross in an effort to determine the possible solutions for the clue for “4 Across.” (Incidentally, a *gross* is a mathematical term for a dozen dozens, or 144.) The first several multiples of 144 are: 144, 288, 432, 576, 720 and 864.

***\*\*\*\*\* Another Opportunity to Exercise your Analytical Reasoning Skills \*\*\*\*\****



Which of these multiples is the correct choice for the clue for “4 Across”?

\*\*\*\*\*

You got it! 😊 Since the only possibilities for a perfect cube are 125 and 343, and since the multiple of 144 in “4 Across” begins with the ending digit of the cubed number, the multiple of a gross must begin with either 5 or 3. An examination of the list of multiples of 144 reveals that the only possible solution is 576.

At this point, you may safely enter 121, 125, and 576 as solutions. Assuming that you have already entered 64 as the solution for “6 Across”, there is only one entry that remains unsolved: “3 Across.” The clue for “2 Down” will provide a valuable piece of information in this regard. Since the leap year is in the twentieth century, its first two digits must be 1 and 9. This means that the two-digit prime number in “3 Across” must begin with 9 and once again our choices are narrowed considerably. There are only five odd numbers between 90 and 100 and only one of these is a prime: 97.

***Math Is Everywhere! Explore and Discover It!***  
***1.2 Math Classics and Applied Reasoning***

***Congratulations! The puzzle has been solved!***

\*\*\*\*\*

While solving this cross-number puzzle may have seemed like a lengthy and somewhat tedious process, in actuality it takes far more time to explain the analytical reasoning processes than it does to actually solve the puzzle. I hope that you enjoyed this exercise in the use of your analytical reasoning skills and problem-solving strategies. You will now have further opportunities to practice these skills and strategies in the *Exercises and Projects for Fun and Profit* below. ***Have fun!***

\*\*\*\*\*

***Exercises and Projects for Fun and Profit***

***Friendly warning and assurance:*** The exercises and projects in this section can tend to be quite time-consuming due to their nature as puzzles; however, please don't be alarmed—the rest of the textbook exercises do not all require this much open-ended effort! ☺

***Note:*** Please see *Helps and Hints for Exercises* on **pp.423–424 in Appendix A.**

***Analytical Reasoning Applications***

1. ***As I was going to St. Ives:*** This classic puzzle originally appeared in the Mother Goose collection of nursery rhymes and gained popularity in the ensuing years. A *google.com* search on this phrase will yield interesting results. There have been many variations of this puzzle down through the centuries; the one that follows has a fairy-tale flavor.

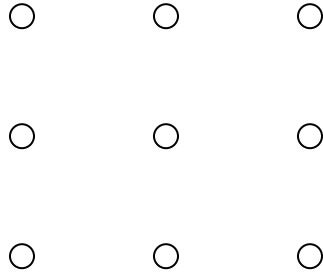
As I was going to the store,  
I met a man with seven doors.  
Every door had seven knobs,  
Every knob had seven locks,  
Every lock had seven keys;  
Doors, knobs, locks, keys,  
How many were going to the store?

Please identify the problem-solving strategies that you use to solve this classic and record your solution. (The strategies are enumerated on p.4 in *Section 1.1* of the text.) There is more than one possible solution—please try to find all logical possibilities.

2. ***Nine Dots: Thinking Outside the Box:*** Our next math classic is a problem that you may have encountered in elementary school or elsewhere. It involves a square containing nine dots as shown below; the objective is to connect all nine dots with four straight, connected line segments

***Math Is Everywhere! Explore and Discover It!***  
***1.2 Math Classics and Applied Reasoning***

without retracing any of the line segments. In other words, you need to connect the nine dots with only four straight line segments without removing your pencil from the paper and without retracing any part of a line segment.



3. ***The Case of the Missing Dollar:*** Another classic math problem is the case of the missing dollar. A variation of the story is as follows:

Three friends went out to dinner one evening for a social visit. The bill for the dinner was exactly \$30 and the three friends decided to split the bill evenly between them. Consequently, each of them paid the waiter \$10. When the waiter brought the bill to the cashier, the cashier realized that an error had been made and that the bill should have only been \$25. She gave the waiter five \$1 bills and asked him to please return the money to the guests. As the waiter returned to the table, he realized that there was no fair way to divide the money between the three guests, so he instead gave each of the guests \$1 and pocketed the remaining \$2 for himself.

This means that each guest actually paid only \$9 for the meal for a total of \$27; the waiter pocketed \$2 for a combined total of \$29. So what happened to the missing \$1?

Please identify the problem-solving strategies that you use to solve this classic and record your solution. (The strategies are enumerated on p.4 in *Section 1.1* of the text.)

4. ***Exact Measures:*** Pouring liquids into containers of various sizes and attempting to get an exact amount of liquid into a container has long been a part of recreational mathematics. The following problem dates back several centuries at least and, as might be expected, requires some persistence and analytical reasoning in order to obtain a solution. This newer version uses a water supply in the form of a faucet with a shut-off handle.

You are provided with a water supply in the form of a faucet with a shut-off handle and two empty jars, one of which holds exactly **three cups** of liquid while the other holds exactly **five cups** of liquid. Neither of the jars has any measurement markings so there is no way to partially fill them, with accuracy, directly from the faucet. Your objective is to obtain **EXACTLY four cups** of water in the five-cup jar by **completely filling the smaller jar from the faucet** when necessary and pouring water from the smaller jar into the larger until you have achieved your

***Math Is Everywhere! Explore and Discover It!***  
***1.2 Math Classics and Applied Reasoning***

objective. You may completely empty the larger jar of water by pouring it on the ground when necessary.

**Note:** You are not allowed to partially fill a jar from the faucet. You may not use any jars or containers other than the two that are specified.

Please identify the problem-solving strategies that you use to solve this classic and record your solution. (The strategies are enumerated on p.4 in *Section 1.1* of the text.)

5. **Exact Measures:** Please follow the instructions for the preceding exercise (#4 above) and use two empty jars, one of which holds exactly **four cups** of liquid while the other holds exactly **seven cups** of liquid. Your objective is to obtain **EXACTLY five cups** of water in the seven-cup jar.

6. **Prize Money:** Two friends have some ten-dollar bills in their pockets and each has less than fifty dollars. Each of them has entered a contest in which the prize is \$100.

The first friend declares: If I win the contest, I'll be three times as rich as you.  
The second replies: Yes, but if I win the contest, I'll be seven times as rich as you.

How much money does each of them have in their pockets?

Please identify the problem-solving strategies that you use to solve this classic and record your solution. (The strategies are enumerated on p.4 in *Section 1.1* of the text.)

7. **Prize Money:** Two friends have some twenty-dollar bills in their pockets and each has less than five hundred dollars. Each of them has entered a contest in which the prize is \$200.

The first friend declares: If I win the contest, I'll be just as rich as you.  
The second replies: Yes, but if I win the contest, I'll be five times as rich as you.

How much money does each of them have in their pockets?

Please identify the problem-solving strategies that you use to solve this classic and record your solution. (The strategies are enumerated on p.4 in *Section 1.1* of the text.)

\*\*\*\*\*

*Creative Projects:*  
*Guards and Prisoners*  
*Cross-number Puzzle*

## GUARDS AND PRISONERS

A similar problem to the fox, goat and cabbage episode involves guards and prisoners. This problem dates back to the nineteenth century and has been recast in various settings. In this particular version, there are a given number of guards and a given number of prisoners who need to cross a river. Their boat holds only two people at a time, and the number of prisoners must never be allowed to outnumber the guards on either side of the river; otherwise, the prisoners will overpower the guards and, well, the story will come to an abrupt end. ☹

*The following rules apply:*

1. For the purposes of this puzzle, prisoners may be left alone on one side of the river or the other and they won't try to escape even though they technically outnumber the guards at that point (since there are no guards present).
2. Also, it is important to note that when a person or persons arrive in the boat, they need to be counted in terms of whether the prisoners will outnumber the guards. For example, if there were one guard and one prisoner on one side of the river and if the boat arrived carrying one prisoner, at that point the two prisoners would outnumber the guard, i.e., the prisoner on the riverbank plus the prisoner in the boat would outnumber the single guard.
3. Someone (either a guard or a prisoner) needs to row the boat across the river during each crossing. Swimming and towing the boat with ropes are not permitted; only one or two people can cross the river in the boat during each crossing.

*Note:* Using colorful markers to represent the guards and prisoners and physically moving them across a boundary line (the river) is a helpful visual strategy. In addition, drawing a diagram and constructing a table are good problem-solving strategies for recording the river crossings.

### Assignments:

For each assignment, please do the following (please make use of the *Helps and Hints for Guards and Prisoners* on pp.424–429 in Appendix A):

1. **Determine how many river crossings (both across and back) it will take** to safely transport all of the guards and prisoners across the river.
2. **List each of the river crossings** that needs to be made, who is in the boat during each crossing and which direction they are traveling (across or back).
3. **Identify and record the problem-solving strategies** that you use to solve this classic.

<i>assignment</i>	<i>number of guards</i>	<i>number of prisoners</i>
<i>Assignment #1</i>	3 guards	3 prisoners
<i>Assignment #2</i>	4 guards	2 prisoners
<i>Assignment #3</i>	3 guards	2 prisoners
<i>Assignment #4</i>	4 guards	3 prisoners

## CROSS-NUMBER PUZZLE

You'll need to use several problem-solving strategies during the process of finding a solution for the following puzzle. Be persistent, and above all else, don't give up!

1		2		3
		4	5	
6	7			
	8	9		10
11				

**Assignments:** For each assignment, use the clues provided to solve the puzzle:

**Note:** *Numbers with leading zeros* are not permissible as answers for the clues.

**Note:** Please make use of the *Helps and Hints for Cross-number Puzzle* on pp.429–431 in Appendix A. It may also be helpful to read the *Cross-number Puzzle Terminology* on pp.449–450 in Appendix B concerning primes, factors, squares, cubes, etc.

<p><b>Assignment #1:</b>  <u>Across</u>                      1. The cube of a positive integer                          (e.g., <math>9^3=9\cdot9\cdot9=729</math>)                      4. A multiple of 111 (e.g., 111, 222, 333, ...)                      6. A number with only two prime factors (and no repeated factors), e.g., <math>34=2\cdot17</math>                      8. The fourth power of a positive integer                          (e.g., <math>7^4=7\cdot7\cdot7\cdot7=2401</math>)                      11. The square of a prime less than 20</p>	<p><u>Down</u>                      1. The square of a positive integer                          (e.g., <math>9^2=9\cdot9=81</math>)                      2. A multiple of 9                      3. A prime less than the square of 6                      5. The cube of a positive integer                      7. The cube of a positive integer                      9. A prime number                      10. An integer that is both a square and a cube</p>
--	--

<p><b>Assignment #2:</b>  <u>Across</u>                      1. The cube of a positive integer                          (e.g., <math>9^3=9\cdot9\cdot9=729</math>)                      4. A multiple of 222 (e.g., 222, 444, 666, ...)                      6. A multiple of a dozen                      8. The cube of a positive integer                      11. The cube of a positive integer</p>	<p><u>Down</u>                      1. The square of a positive integer                          (e.g., <math>9^2=9\cdot9=81</math>)                      2. A multiple of 9                      3. Twice the value of 10 Down                      5. The square of a positive integer                      7. The square of a prime less than 30                      9. A number divisible by 6                      10. A prime number greater than 32</p>
--	---

**Math Is Everywhere! Explore and Discover It!**  
**1.2 Math Classics and Applied Reasoning**

<p><b>Assignment #3:</b>  <u>Across</u>            1. The square of a positive integer                (e.g., <math>9^2=9\cdot9=81</math>)            4. The square of a positive integer            6. The fourth power of a positive integer                (e.g., <math>7^4=7\cdot7\cdot7\cdot7=2401</math>)            8. The cube of a positive integer            11. The square of a positive integer</p>	<p><u>Down</u>            1. The square of a prime number            2. A number that is both a square and a cube            3. A prime number less than 30            5. A multiple of 17            7. The fourth power of a positive integer            9. One-fourth the value of 2 <i>Down</i>            10. A prime number between 75 and 100</p>
---	--

<p><b>Assignment #4:</b>  <u>Across</u>            1. The fourth power of a positive integer                (e.g., <math>7^4=7\cdot7\cdot7\cdot7=2401</math>)            4. The square of a positive integer                (e.g., <math>9^2=9\cdot9=81</math>)            6. A prime number between 50 and 60            8. The fourth power of a positive integer            11. The number of years in a century</p>	<p><u>Down</u>            1. The square of a positive integer            2. A multiple of 31            3. The square of a positive integer            5. The cube of a positive integer            7. The number of degrees in a circle            9. A number that is evenly divisible by the                sum of its digits            10. A prime number greater than 17</p>
---	--

\*\*\*\*\*

*Answers to Questions in this Section*

**p.10 Crossing the River: The Story of the Fox, the Goat and the Cabbage**

**Strategy #3: Eliminate the impossible; then only the possible remains.**

\*\*\*\*\* *Another Opportunity to Exercise your Analytical Reasoning Skills* \*\*\*\*\*

It actually makes no difference whether the farmer takes the fox or the cabbage next. If he takes the fox across next and then brings back the goat, he can then leave the goat on the original side of the river while he takes the cabbage across the river to join the fox.

If he takes the cabbage across next and then brings back the goat, he can then leave the goat on the original side of the river while he takes the fox across the river to join the cabbage. In either case, the fox and the cabbage are safe together and nothing gets inadvertently eaten...

Lastly, of course, the farmer can return to get the goat and bring the goat across the river, thus safely conveying all of his possessions across the river.