

2.1 Deductive and Inductive Reasoning:

“A good puzzle should demand the exercise of our best wit and ingenuity, and although a knowledge of mathematics and of logic are often of great service in the solution of these things, yet it sometimes happens that a kind of natural cunning and sagacity is of considerable value.” –Henry Dudeney, British puzzlist

Introduction

Welcome to the wonderful world of patterns and reasoning! This is a very enjoyable facet of mathematics where there are plenty of opportunities to exercise your creativity and resourcefulness and have fun in the process. I believe that there is something inherent in our human nature that delights in solving puzzles and searching for solutions; this section invites you to enter into this process and it presents some basic ideas that will carry on throughout the text.

Riddles: Ancient and Intriguing

We'll begin with a look at an interesting class of reasoning exercises known as “*riddles*.” Dictionaries generally define a “*riddle*” as:

1. a question or statement so framed as to exercise one's ingenuity in answering it or discovering its meaning
2. a puzzling question, statement or problem, usually presented as a game or pastime
3. a puzzling or inexplicable thing or person

??? *Hmm-m-m.* ???



Riddles are one of the oldest forms of puzzles and have a long and illustrious history. From Aristotle to Shakespeare to Tolkein, authors through the ages have used riddles to enliven their storybook characters and presentations. Riddles beckon the reader to unravel the mystery contained within and use their creative thinking abilities to find a solution. Unlike the traditional math “problem” with its highly structured presentation and limited scope, riddles leave the door wide open for creativity, free-form association and lateral thinking, and invite everyone to enter in and join the fun without regard to their “math skills.” ☺



Corny, lame or inspired? You be the judge.



Here are some examples to get us started (if you should need them, hints are included at the end of this section on p.43 immediately preceding the *Exercises and Projects for Fun and Profit*):

1. Why did the young man name his new puppies Biology, Chemistry and Physics?
2. What has a face without eyes, and hands without arms?
3. What did one half say to the other half as they were walking down the road together?
4. What is it that leaves but goes nowhere?
5. What is the favorite food of an Italian lawyer who is on a diet? (You need to know some legal terminology for this one; it also helps if you're Italian or like to eat various Italian foods....)

Now, wasn't that fun? I hope that you've answered, "Yes." If you thought that any of the above were corny or lame, I take full responsibility since I created them; nonetheless, I hope that you sensed your creative thinking processes slip into gear and the accompanying surge of adrenalin as your mind began drawing on inner resources in its search for solutions. This is one of the wonderful aspects of mathematical reasoning—it energizes one's mental systems and networks and refreshes the whole body.

Aha! Eureka! I've got it!

While you were probably not consciously aware of it, you began to use your mathematical reasoning skills in the process of solving the riddles. You most likely:

- ***made some initial guesses*** (we formally call these "conjectures" in mathematics),
- ***tested your guesses*** and realized that they didn't quite meet the conditions of the riddle (experimentation and verification), and
- hopefully ***arrived later at a solution*** (we call this the "***Eureka!***" ***moment*** if you're in a particularly Greek mood or the "***Aha!***" ***moment*** otherwise; more often, you might just yell out, "***I've got it!***").



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In any case, the discovery moment in the mathematical reasoning process is a wonderful experience—it distinctly and uniquely satisfies that inner drive to find a solution and builds an inner confidence that may then be applied to other riddles and conundrums.

Bilbo's Challenge

Let's look at some further examples of riddles and keep the creative machinery in motion. The next few are similar to those from J.R.R. Tolkien's *The Hobbit* in the famous encounter between Bilbo Baggins and old Gollum (if you should need them, hints are included at the end of the section on p.43 immediately preceding the *Exercises and Projects for Fun and Profit*):

1. "Its bottom is hidden,
Its head hard to find.
Its mouth always open,
As it speaks to mankind."



2. "Leaves no footprints,
Nary a track,
Travels the world
Around and back."

3. "No light of its own,
A guide for men;
Mover of waters,
Within a sailor's ken."

4. "Formless yet strong,
With time on its side;
Ever consuming,
Never satisfied."

5. "I exist at war's outset,
In the midst of battle,
And at the end of all conflict."



Invigorating, isn't it? You've just experienced one of the joys of mathematics—the process of searching for connections and reasons and solutions.

That is how mathematics should always be experienced—it's a true adventure with discoveries waiting around every bend.

This course has been designed to lead you on such an adventure, one in which I trust that you will find great delight and one that will draw you into a new and exciting relationship with mathematics. Let's proceed!

Searching for Patterns: A Sherlock Holmes Mystery Case

Looking again to literature, we find that Sherlock Holmes, that most famous of detectives, often applied mathematical reasoning in solving his cases. There is a particularly appropriate example in “*The Adventure of the Creeping Man*” in which Holmes analyzes the mathematical pattern in the occurrences of a series of unusual events. As he delved into the details of the case, the following facts emerged:



The unusual events occurred on the dates:

July 2, July 11, July 20, July 29, August 7, August 16, August 26, and September 5.

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Using your mathematical reasoning skills, can you determine the most likely date of the next occurrence? Please pause for a moment and give it a try—*this is an opportunity to exercise your powers of observation and begin to build some confidence in your reasoning ability.*

Congratulations! Sherlock would have been proud of you! This is exactly what Holmes induced and his keen observation led to the solution of this bizarre mystery. (Answers with which you may confirm your solutions are provided at the end of this section on p.47 immediately after the *Exercises and Projects for Fun and Profit.*)

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While the above example was rather elementary, the dates in the *Granada Television* version of this mystery were modified to provide a bit more mathematical interest and complexity:

The unusual events occurred on the dates: September 15, September 26, October 5, October 12, October 17, and October 20.

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Using your reasoning skills once again, can you determine the most likely date of the next occurrence and the reason for Holmes' sense of urgency on October 20? This pattern is slightly less obvious than the original one, but I'm sure that you are capable of meeting the challenge. ***Again, this pause to use your powers of observation is designed to build your confidence, so please give it a try!***

Congratulations once again! Give yourself a pat on the back and be encouraged! (Answers with which you may confirm your solutions are provided at the end of this section on p.47 immediately after the *Exercises and Projects for Fun and Profit.*)

Recognizing Pattern Discrepancies

In actuality, the dates in the film were shown by number as follows:



21, 15, 26, 5, 12, 17, and 20.

The first date in the list, 21, was intentionally omitted from the previous example to eliminate the confusion that it introduces when attempting to determine a pattern in the dates. It appears that an inadvertent error was made at the television studio when producing this film and your next challenge is to determine what error may have been made in listing the first date.

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What should the first date have been in order to properly conform to the pattern of the rest of the dates?

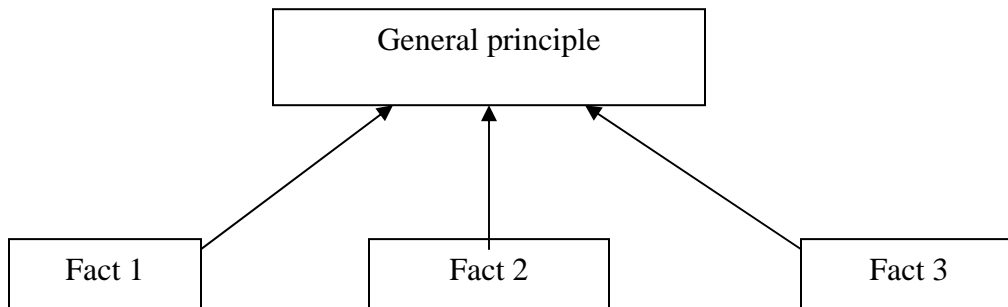
Once more, please pause to work this out; you'll continue to build that invaluable sense of confidence in your mathematical reasoning abilities. ☺

I trust that you successfully found the correct first date and are now experiencing that inner satisfaction that accompanies solving a puzzle. (Answers with which you may confirm your solutions are provided at the end of this section on p.47 immediately after the *Exercises and Projects for Fun and Profit*.)

Inductive Reasoning: Postal Deliveries

In the above example, Sherlock Holmes made use of what is called **inductive reasoning**, a form of reasoning that uses a number of specific facts or instances to form a general rule that applies to all of the given facts. We often use this form of reasoning in our daily lives although we may not be consciously aware of the process.

Consider, for example, the daily U.S. postal delivery. Most of us have probably observed the delivery patterns in our neighborhoods and have determined, on the basis of our casual and repeated observations, that the mail is delivered between certain hours of the day. In my neighborhood, for example, the mail consistently arrives between noon and 2:00 p.m. Based on my repeated observations, I feel justified in declaring that the mail *always* arrives between noon and 2:00 p.m. This is the **inductive reasoning** process in action: forming a general rule from a set of specific instances. You might think of it as “**bottoms-up**” logic



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There is an inherent amount of risk involved in this type of reasoning; in other words, this reasoning may lead to faulty conclusions. ***Do you see what makes this reasoning risky?*** (Answer is provided at the end of this section on p.47 immediately after the *Exercises and Projects for Fun and Profit.*)

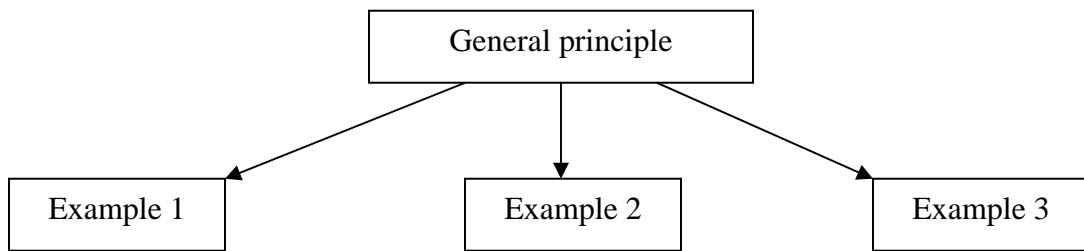
Deductive Reasoning: Paycheck Regularity (Thank goodness!)

Despite the inherent risk involved, mathematical reasoning is a very valuable resource, one that may be used to purchase delightful results. On the other side of the reasoning coin is ***deductive reasoning***, or “***top-down***” logic, the process of using an established general rule to determine specific instances. For example, if you know that your employer always issues paychecks every two weeks on Fridays, then you can look at the calendar and determine all of your specific pay dates for the coming month or year, thus making it easier to determine when you’ll have enough money to go out on that expensive date that you’ve been planning. Ah, a delightful thought!

On a more mathematical note, if we begin with the principle that every prime number has only two positive whole number factors, the prime number itself and 1, then we can specifically construct a list of prime numbers.

2, 3, 5, 7, 11, 13, ... (1 is not technically considered a *prime*.)

Again, this is an example of ***deductive reasoning*** in which we work from a general rule to construct a set of specific instances of the rule.



A Bit of Latin for Those Interested: The “ductive” Types of Reasoning

Inductive and deductive reasoning both have as their etymological root the Latin verb ***ducto***, ***which means “to lead.”*** Combining this root word with various Latin prefixes produces a number of words in our English language:

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<i>Latin prefix</i>	<i>meaning</i>	<i>English word</i>	<i>meaning</i>
in	in or into	induct	to lead into
de	out from	deduct	to lead out from
con	with	conduct	to lead with
ab	away from	abduct	to lead away from

In the mathematical context,

- ***inductive reasoning*** observes a set of diverse instances and ***leads into*** a general conclusion
- ***deductive reasoning*** observes a general rule or principle and ***leads out*** from there to establish a set of instances based on the rule.

Divisibility: An Illustration

In order to solidify our grasp of these types of reasoning, we will now take a look at some basic principles involving whole numbers and their divisors.

As we all know, every *even* number is evenly divisible by two (with no remainder); you might call this a general rule. On that basis, then, you could go on to determine whether any specific number were *even*. For example, the number 39 when divided by 2 produces a quotient of 19 and a remainder of 1; consequently, we conclude that 39 is not *even* since it does not comply with the above rule. What type of reasoning is this: inductive or deductive? (***Answer: deductive, since we are using a general rule to establish a specific instance.***)

Now consider the following multiples of 5:

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, ...

- If you will observe carefully, you will notice a pattern that appears in the ending digits of these numbers. What is this pattern?
- Based on your observation, what can you infer about divisibility by 5? What type of reasoning is this: inductive or deductive?

Very good! You have noticed (I'm sure) the fact that all listed multiples of five have ending digits of either 0 or 5. Based on this observation, you concluded (I hope) that any number that ends in either 0 or 5 is a multiple of 5 and consequently is evenly divisible by 5; this is ***inductive reasoning—using specific examples to form a general rule or principle.***

The Slippery Slope of Inductive Reasoning

Consider numbers of the form $n^2 + 1$ where n is an even positive integer:

$\frac{n}{2}$	$\frac{n^2 + 1}{5}$
4	17
6	37



Since the first three values for $n^2 + 1$ turned out to be *prime* numbers, it would appear as if $n^2 + 1$ will *always* turn out to be a *prime* number.

It is often the case that three instances do establish a pattern that holds true in all further instances; however, as noted earlier, there is an element of risk involved with inductive reasoning.

Even though we observe a number of instances that seem to follow a general rule, there may be some as yet unobserved instances that, if observed, would nullify our induced rule. In this case, the next value for n produces a value for $n^2 + 1$ that is *not prime* ($n=8$ yields $n^2 + 1=65$); an instance such as this is referred to as a ***counterexample***. And the house that was built upon the sand fell with a great crash...

“Oh, dear!” you may be thinking, “Whatever will we do if we can’t depend on our inductive reasoning process? Will all be lost and will chaos ensue?” Not to worry. While our inductive reasoning may at times lead us astray, mathematicians demand that every conjecture and every line of reasoning be formally proved before being accepted as fact. We won’t pursue ***mathematical proofs*** in this text, but rest assured that they guarantee that mathematics rests on a solid foundation.

Analytical Thinking in Action

As you are already discovering, this course is not "math as usual" and you'll embark on many mini-adventures as we proceed through the semester. In addition, some of the *Exercises and Projects for Fun and Profit* are not usual, either, in the sense that there may be no specific "cookie cutter" examples to follow. In particular, some of the exercises include questions that require you to explore and investigate using your own mathematical reasoning skills without the benefit of exact patterns to follow.

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Making a concerted effort to answer the questions is most important whether or not your answers are correct. So just go ahead and make some good guesses or conjectures--it's the effort at analytical and creative thinking that is of value here--not so much the final results.

By the way, it's perfectly acceptable to make incorrect (but creative) guesses in mathematics--in fact, that is what most mathematicians do when faced with new and unusual problems. Eventually, they refine their incorrect (actually creative) guesses until in some cases they arrive at a "right" answer.

If you think more about it, you may realize that this is exactly what we do in our personal life-decision-making processes all the time. We make our best guess as to what the "correct" decision is regarding a particular situation and then often discover that the "correct" decision wasn't the best decision; so we modify that and make a revised decision that hopefully serves the situation more effectively.

This process of trial and error, or guess and revise, is a basic pattern of mathematical reasoning.

Rarely does anyone guess the "perfect solution" on their first attempt and many attempts are sometimes required before a satisfactory result is obtained. (Related trivia question: How many times did Thomas Edison try to make a light bulb before he got one to work successfully? ☺)

In any event, please don't feel intimidated by problems that don't seem to yield immediate solutions--this is part of the nature of mathematics (and life); but do be encouraged to keep trying and making guesses until something satisfactory emerges or, in some cases, until you simply give up for a while--and that's ok, too.

I welcome with open arms your guesses, educated or wild, and your conjectures, partial or complete, knowing that in the process of forming them you are using your creative thinking skills and are developing an awareness of the nature of mathematical reasoning.

2.1 Deductive and Inductive Reasoning

Occasionally you will encounter a "Eureka!" moment that will make you realize that your struggles and efforts have been worthwhile. These are moments to be treasured--in fact, you may remember these moments years later, long after much of what else you learned during that period is forgotten.



These moments, which are only encountered through persistent effort and a determined resolve not to easily give up on trying to find solutions or understand a concept, are memorable because they are intensely personal. In them you overcome a seemingly insurmountable difficulty or dilemma and in the process build confidence in your analytical and problem-solving abilities and even, at times, in your own self-worth.

In conclusion, then, don't be dismayed at problems or exercises that at first don't yield solutions or that don't have "cookie-cutter" examples to guide you. Instead, be diligent and persistent in making guesses and conjectures, be willing to give up for a while and come back later (your subconscious mind is a great ally), and be assured that it's ok to be "wrong" as long as you have made a good effort.

One of my goals is to help you develop your mathematical reasoning skills; by the end of your experiences with this textbook I hope that you will look back on this course of study as a very positive experience in that regard!

Insights and Conclusion

In this section we examined *top-down* and *bottoms-up* logic: **deductive reasoning** leads out from a general principle to establish specific instances of the principle whereas **inductive reasoning** begins with a set of facts or examples and leads into the development of a general principle. Through an example involving prime numbers we came to the realization that **inductive reasoning, the bottoms-up variety, is inherently risky and prone to error**. This, in turn, led to a realization of the need for an assurance of certainty; fortunately, mathematical proofs provide this assurance. The world would be a very shaky place if we couldn't rely on anything!

Underlying these forms of reasoning is the realm of creative thinking—the very heart of mathematics. We began with some riddles and a mystery case to engage your creative skills and to illustrate how enjoyable and invigorating it is to exercise your reasoning powers. The realm of creative thinking underlies all other mathematical activity and is the seedbed for many rich and fruitful mathematical ideas.

I truly hope that throughout the rest of this course you come to fully enjoy the creative process and to build confidence in your mathematical reasoning skills—it's a wonderful and delightful experience!



Hints for Questions and Activities in This Section

p.33 Corny, lame or inspired? You be the judge. (“Correct” answers provided on p.47 following the Exercises and Projects for Fun and Profit.)

1. Think about the various types of dog breeds.
2. Common household item, although styles have changed in the past decade.
3. Lots of acceptable answers for this one.
4. Grows in yards, parks, farms, etc.
5. Required legal term: tort

p.34 Bilbo’s Challenge

1. Canoeing, anyone?
2. Driftwood knows all about this.
3. Think of the beach.
4. Winter nights.
5. Be literal.

2.1 Deductive and Inductive Reasoning

Exercises and Projects for Fun and Profit

Exercises in Creativity

*Hey-diddle-diddle, the cat told a riddle,
The cow fell down in a swoon.
The little dog stared while the hamster glared
And the fish flew away on a balloon.*



Get in a relaxed and creative mood, and try to solve the following riddles:

1. What has a head like a cat, whiskers like a cat, feet like a cat, tail like a cat, but is not a cat?
2. It has cities, but no houses,
Mountains, but no trees.
It has oceans, but no water
And rivers, but no fish.
What is it?
3. What is black and white and red all over?
4. Much higher than the trees I am,
My bulk exceeds a house;
And yet I seem unto most men
No larger than a mouse.
5. What is that which dwells once in earth, twice in heaven, and lives in the middle of men?

In the next set of exercises you will have an opportunity to apply your analytical thinking and reasoning skills.

I wish you all the very best of success as you work your way through the exercises and projects! Those "Eureka!" moments are priceless... ☺

Trial and error combined with some analytical reasoning will lead you to success in the exercises that follow. In fact, all of these can be solved completely by trial and error and the use of a calculator; however, you can save a good deal of time and effort by using your analytical reasoning skills to shorten the testing process.

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- 6. **Four by five:** Using each of the digits 5, 3, 6, and 7 exactly once (i.e., no duplicates allowed) to construct each four-digit number, construct as many four-digit numbers as possible that are evenly divisible by 5 (i.e., divisible by 5 with no remainder). As examples, 3675 and 3765 are two such numbers; please find all other solutions.
- 7. **Four by six:** Using each of the digits 5, 3, 6, and 7 exactly once (i.e., no duplicates allowed) to construct each four-digit number, construct as many four-digit numbers as possible that are evenly divisible by 6 (i.e., divisible by 6 with no remainder).
- 8. **Four by seven:** Using each of the digits 5, 3, 6, and 7 exactly once (i.e., no duplicates allowed) to construct each four-digit number, construct as many four-digit numbers as possible that are evenly divisible by 7 (i.e., divisible by 7 with no remainder).

The next two exercises require you to use your powers of observation, inductive reasoning, and analytical thinking; if that sounds too intimidating, pretend that you didn't read it and go on to the exercises anyway...

- 9. **By four?** Can you determine how to tell if a number is evenly divisible by 4 without actually dividing the number by 4? It will help to observe some examples of numbers that are divisible by 4 in various categories such as numbers between 1 and 100, numbers between 100 and 200, and numbers between 200 and 300 to get a better feel for what is involved. Look for patterns, even very general ones—anything that repeats. This repetition is a clue that will lead you to the secret involved in the rule. Press on Mr. Holmes!
- 10. **By six?** Can you determine how to tell if a number is evenly divisible by 6 without actually dividing the number by 6? State a possible test for divisibility by 6. (Hint: this is a two-part test involving the divisibility tests for two of the factors of 6. A number is evenly divisible by 3 if the sum of its digits is evenly divisible by 3, e.g., 189 is evenly divisible by 3 since $1+8+9=18$ and 18 is evenly divisible by 3.)

This next exercise may be done by brute force (lots of trial and error) or you may reason your way to a shorter route to a solution. (It's always quite satisfying when you see a way to shorten your work...)

- 11. **Four by four:** Using each of the digits 5, 3, 6, and 7 exactly once (i.e., no duplicates allowed) to construct each four-digit number, construct as many four-digit numbers as possible that are evenly divisible by 4. (If you haven't yet answered #9 above, you may do so now or you may read the *Rule for Divisibility by 4* provided on p.48 at the end of this section.)

*Creative Project:
A Riddling Good Time*

A RIDDLING GOOD TIME

As noted in this section, solving riddles requires mathematical thinking. This project is designed to provide an enjoyable way for you to enter into this process and exercise your creativity as well.

Assignment:

Create your own original riddles (minimum of three riddles but by all means do more if you get on a roll...). You might choose a topical theme as a focus or you might take an open-ended approach. Riddles need to be clean and wholesome—nothing suggestive or off-color, please. Please include your answers as well.

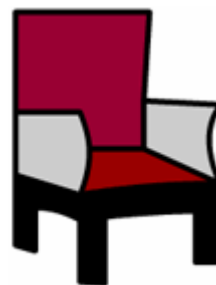
Note: Riddles have an official and authoritative right to be “corny” and/or “lame,” so please don’t feel intimidated by this exercise in creativity. Your riddles will be just as valid as any others and, since there are no rules for what riddles should or shouldn’t be, you needn’t worry about measuring up to some predetermined standard. It is the effort in creative thinking that lies at the heart of this exercise—not the quality of the results. Please be persistent—you may pleasantly surprise yourself with your own hidden creativity!



Since this is a very open-structured project with no particular place to start, you may feel at somewhat of a loss at first. If so, you might begin by selecting an everyday object with which you are familiar, listing all of the object’s features, and then creating a riddle that relates to a feature of interest.

Example:

- Everyday object: chair
- Features: A chair has two arms (sometimes), four legs, a seat, a back, some rungs, a cushion (sometimes), etc.
- Possible riddle: What has legs but cannot walk, and a back but cannot bend?



Simplistic? Yes, but that’s ok.

Creative? Definitely!

Original? Yes, I did this all by myself. 😊

Personally satisfying? Yes! There are intangible benefits connected to using your creative thinking powers to invent something new and unique. These benefits are both wide-ranging and deep, and are more easily experienced than described. You’ll see what I mean as you engage yourself in this project...

2.1 Deductive and Inductive Reasoning

Answers to Riddles and Questions in this Section

p.33 Corny, lame or inspired?

1. Because they were all Labs (i.e., Laboradors)
2. a clock
3. “We’d better be careful or we might fall into a whole.”
4. a tree
5. tort-a-leany (“tort” is a legal term; *tortellini* is a type of Italian pasta)

p.34 Bilbo’s challenge

1. a river
2. ocean currents (or the wind)
3. the moon
4. fire
5. the letter “t”

Sherlock Holmes case

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Most of the dates are nine days after their respective predecessors; the last two dates are ten days after their predecessors. Consequently, the next date would logically be either September 14 (nine days later) or September 15 (ten days later). You could argue in favor of either choice. ..

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The successive differences between dates follow a distinctive pattern:

11 days, 9 days, 7 days, 5 days, 3 days.

The next logical difference in this sequence is 1 day and the next date, therefore, would be October 21 (hence Mr. Holmes’ sense of urgency).

*p.37 *** One More Opportunity to Build your Mathematical Confidence ****

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The first date should have been 2, not 21, so that the sequence would have read:

2, 15, 26, 5, 12, 17, 20.

The successive differences between the dates then follow a nice, orderly progression:

13 days, 11 days, 9 days, 7 days, 5 days, 3 days

Perhaps the “21” was a typographical error in the film editing and production process...

p.38 Do you see what makes this reasoning risky?

The fact that we observe a number of examples that seemingly follow a pattern does not, in and of itself, guarantee that the pattern will always continue; there may be an instance that we have not observed that nullifies the apparent pattern.

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p.45 Rule for divisibility by 4

If the ending two digits of a number are evenly divisible by 4, then the given number is evenly divisible by 4.

Examples:

- 5736 has 36 as its final two digits. Since 36 is evenly divisible by 4, the original number, 5736, is also evenly divisible by 4.
- 9364 has 64 as its final two digits. Since 64 is evenly divisible by 4, the original number, 9364, is also evenly divisible by 4.

This rule can be induced by examining the multiples of four from 1-300 and noticing that the ending two digits repeat, beginning at 104. Hence, this pattern will continue and we need only to focus on the ending two digits in a given number. Pretty simple, isn't it?

<i>Multiples of 4:</i>	4,	8,	12,	16,	20,	24,	...	100,
	104,	108,	112,	116,	120,	124,	...	200,
	204,	208,	212,	216,	220,	224,	...	300,

Another way of thinking about this is shown in the example below:

Example: 524 = 500 + 24

Every multiple of 100 is evenly divisible by 4, so 500 is evenly divisible by 4. The question of essence, then, is only whether the number formed by the ending two digits is evenly divisible by 4. In this case, 24 is evenly divisible by 4, so the number 524 must also be evenly divisible by 4.

As additional verification, since both numbers on the right are evenly divisible by 4, we could factor out a 4:

$$524 = 4 \cdot (125 + 6)$$

Then 524 is clearly a multiple of 4 and therefore must be evenly divisible by 4.

Example: 386 = 300 + 86

Every multiple of 100 is evenly divisible by 4, so 300 is evenly divisible by 4. The question of essence, then, is only whether the number formed by the ending two digits is evenly divisible by 4. In this case, 86 is NOT evenly divisible by 4, so the number 386 must NOT be divisible by 4.

This same line of reasoning applies to all numbers, no matter how large.